

Riemann Sums Activity

Overview

Riemann Sums can be used to approximate the definite integral of a function. A common application of these sums is to estimate the area under a curve. This activity introduces students to Riemann Sums and illustrates how they can be used to approximate the area under a curve.

Objectives

Students will:

- Connect the definite integral with the area under a curve
- Understand how Riemann Sums are calculated
- Be able to approximate the area under a curve with a Riemann Sum

Standards (TEKS)

Precalculus: P.3A, B, C, D, P.4D

Calculus Standards

Activity

- 1) Open and run the VI.
- 2) By default, this VI uses $f(x) = x^2$, i.e. **Function** is set to x^2 . If necessary, reset **Function** to x^2 .
- 3) Make sure **Start** and **End** are set to 0 and 6, respectively.

RiemannSum.vi computes the definite integral between **Start** and **End**. For example, the value of **Definite Integral**, $\int_{Start}^{End} f(x) dx$, is $\int_0^6 x^2 dx = 72$, with the default values. This value is the area under the curve between 0 and 6.

The VI also computes two Riemann Sums. One uses the function values at the left endpoints of each subinterval. The other uses the function values at the right end points. The subintervals are **Delta x** in length.

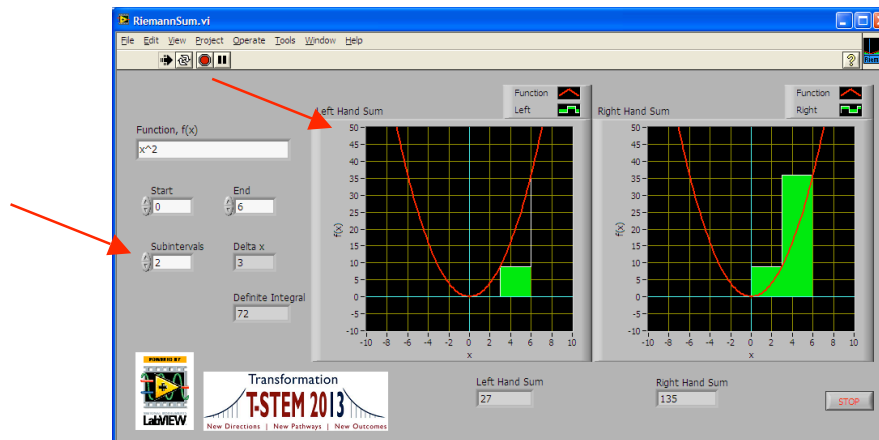
This may sound complicated. However, these Riemann sums are simply the sum of the areas of some rectangles. The width of each rectangle is **Delta x**, and you get the height of each rectangle by evaluating the function at different spots. You'll see this shortly.

- 4) Set the maximum value on the $f(x)$ -axis of the graphs to 50, by double-clicking on the value and setting it to 50.
- 5) Set **Subintervals** to 1. Since the interval is 6 units long, **Delta x** will equal 6.

In the Right Hand Sum, the right endpoint is 6, and $f(6) = 36$. The area of the one rectangle is $6 * 36 = 216$. In the Left Hand Sum, the left endpoint is 0, and $f(0) = 0$. The area of that “one” rectangle is $6 * 0 = 0$.

So far, we have grossly over and underestimated the actual area.

6) Next, set **Subintervals** to 2 (**Delta x** will equal 3.)

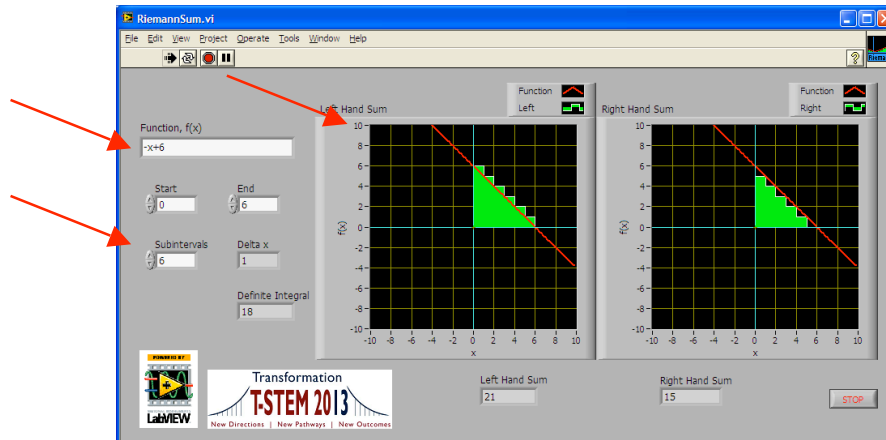


Now, there are, of course, two rectangles for each sum. The length of the subintervals, i.e. the width of the rectangles, is 3, and the heights are determined using either the left or right endpoints. In this case, the Left and Right Hand Sums are easy enough to compute. For example,

$$\begin{aligned}
 \text{Left Hand Sum} &= 3 * f(0) + 3 * f(3) \\
 &= 3 * 0 + 3 * 9 \\
 &= 27
 \end{aligned}$$

- 7) Confirm the Right Hand Sum has been computed correctly.
- 8) Now, set **Subintervals** to 3, 6, and 12.
 - a) Notice that each step of the way, there are more rectangles and the estimations are narrowing in on the value of the definite integral.
 - b) What is the value of **Delta x** in each case? Do these values make sense?

- 9) Change **Function** to $-x+6$, **Subintervals** to 6, and change the graphs so that the largest $f(x)$ value displayed is 10.



- 10) Evaluate the definite integral and find the Left and Right Hand Sums for the following functions, over the specified interval, with the given number of subintervals.

- $f(x) = x^3$, $0 \leq x \leq 2$, 4
- $f(x) = \sqrt{x}$, $0 \leq x \leq 10$, 5 (Hint: Use sqrt(x) for \sqrt{x})
- $f(x) = \frac{1}{x}$, $1 \leq x \leq 3$, 10
- $f(x) = -x^2 + 4$, $-2 \leq x \leq 2$, 8