Transformation
T-STEM 2013
New Directions | New Pathways | New Outcomes

Pyramids Activity

Overview
In this activity, students develop their understanding of surface area and volume of pyramids. Students will calculate the surface area and volume. The Pyramids virtual instrument, VI, will be used in performing and checking calculations as well as building a visual and conceptual understanding of surface area and volume.

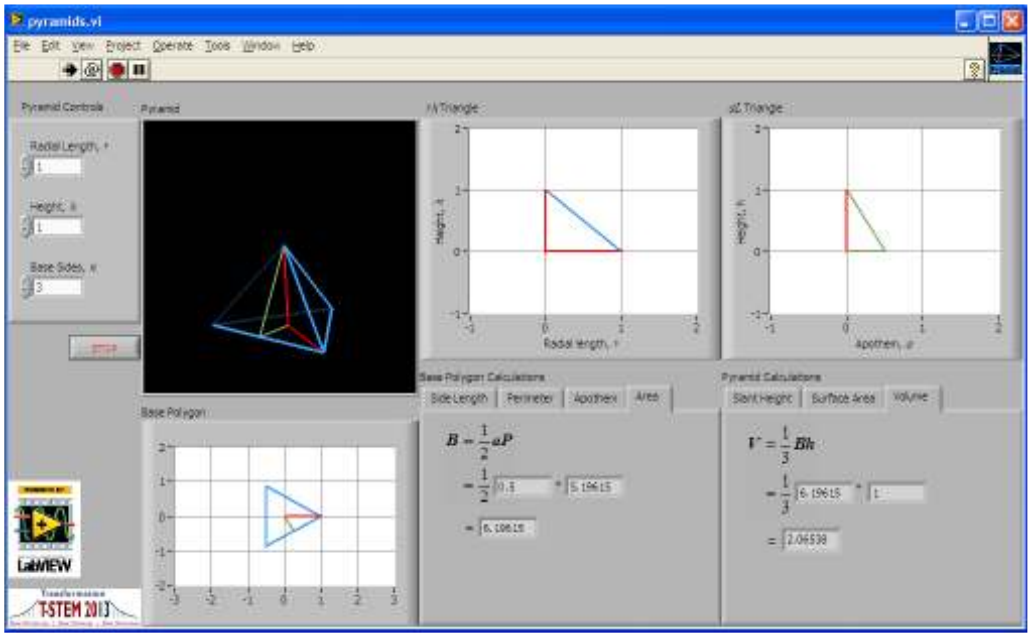
Objectives
Students will be able to:

- Describe the difference between surface area and volume.
- Calculate the surface area and volume of a pyramid.

Standards (TEKS)
Geometry 2AB, 3B, 4, 5AB, 6ABC, 8AD, 9B

Activity
The **surface area** and **volume** of a **pyramid** both depend on the shape and size of the base of the pyramid. The surface area deals with the outside of the pyramid, while the volume deals with the inside. If you were painting the outside of a pyramid, you would want to know the surface area. If you were filling a pyramid with sand, you would want to know the volume.

1) Open and run the Pyramids virtual instrument, VI.



The Pyramids VI has multiple views of the pyramid and several calculations. In the upper left, you will see controls for the radial length of the base, *r*, the height of the pyramid, *h*, and the number of sides of the base, *n*. The 3D graph shows the pyramid in blue and other items in red and green.

There are three 2D graphs displaying the base and two cross sections of the pyramid. Then, there are two sets of tabs – one for calculations related to the base and the other for calculating the surface area and volume of the pyramid.

2) Play with control values to see how the pyramid changes.
    a) What happens when *r* changes?
    b) What happens when *h* changes?
    c) What happens when *n* changes?

As the pyramid changes, so does its surface area and volume. Using the VI, let's take a look at how volume is calculated.

3) Click on the **Volume** tab and make note of the formula for the volume of a pyramid.

    V = _____

4) Set the control values to *r* = 1, *h* = 1, *n* = 3.
5) What shape is the base of the pyramid?
6) Assuming the lengths are measured in feet, what is the area of the base of the pyramid?
7) Assuming the lengths are measured in feet, what is the volume of the pyramid?
8) What happens to volume if you double the height?
9) What happens to volume if you double the radial length?

Let's take a close look at how volume is calculated. To find the volume, we need to know the area of the base. The base is a regular polygon. Its area, *B*, can be found using the formula $B = \frac{1}{2}aP$, where a is the length of the apothem and *P* is the perimeter of the base. To find the base, then, we need to find both the length of the apothem and the perimeter.

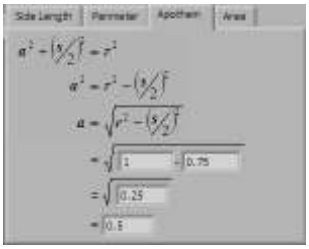
Again, to find the volume,
You need the area of base.
To find the area of the base,
You need the apothem and perimeter.

10) Reset the control values to *r* = 1, *h* = 1, *n* = 3.
11) Click on the **Perimeter** tab.

a) Make note of the formula for perimeter. *P* = _____
b) What is the perimeter of the base?

12) Click on the **Side Length** tab.
a) Note that trigonometry is used to find the length of the side.
b) What is the length of one side of the base polygon?
c) Confirm this is the same value used on the **Perimeter** tab.

Next, you will find *a*, the length of the apothem. The apothem forms a right triangle with the radial length and a side of the base. The apothem is the green line in the graph of the Base Polygon. As you can see on the apothem tab, the Pythagorean theorem can be used to find *a*.



13) What is the length of the apothem?

14) Use the formula for *B*, $B = \frac{1}{2}aP$, to calculate the area of the base.
a) *B* = $\frac{1}{2}$*_____*_____ = _____
b) Check your answer with the information in the **Area** tab.

15) Using the value you have for *B*, calculate the volume of the pyramid.
a) *V* = $\frac{1}{3}$*_____*_____ = _____
b) Check your answer with the information in the **Volume** tab.

16) Set the control values to *r* = 1.1547, *h* = 1, *n* = 3.

17) Find the volume of the new pyramid.
a) Use VI to find *s*.
b) Find *P* = _____*_____ = _____
c) Use VI to find *a*.
d) Find *B* = $\frac{1}{2}$*_____*_____ = _____
e) Check your answer with the VI.
f) Find *V* = $\frac{1}{3}$*_____*_____ = _____
g) Check your answer with the VI.

18) Find volume of a pyramid with a triangular base if the base perimeter is 18 m, the apothem is 1.732 m, and the height is 4 m.

19) Find the volume of a pyramid if the area of the base is 7 in^2 and the height is 3 in.

20) Set the control values to *r* = 1, *h* = 1, *n* = 4.

21) What is the shape of the base?

22) Find the volume of the new pyramid.
a) Use VI to find *s*.
b) Find *P* = _____*_____ = _____
c) Use VI to find *a*.

d) Find $B = \frac{1}{2}$*_____*_____ = _____

e) Check your answer with the VI.

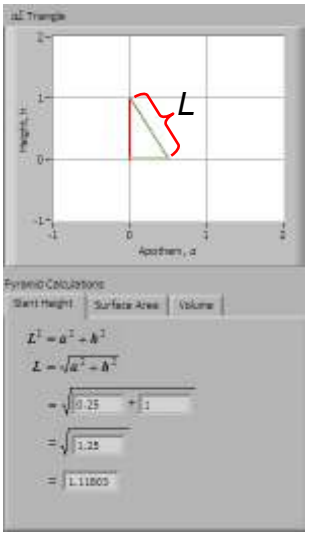
f) Find $V = \frac{1}{3}$*_____*_____ = _____

g) Check your answer with the VI.

23)What does the volume tell you about a pyramid?

Next, let's examine surface area. The first thing to know is there are two different surface areas when it comes to pyramids. There is **lateral surface area** and **total surface area**. Lateral surface area is sum of the areas of the sides excluding the base. Total surface area is the sum of the area of all of the sides including the base.

The lateral surface area is the one-half the product of the perimeter of the base and something known as the slant height, or $\frac{1}{2}PL$, where *P* is the perimeter and *L* is the slant height.



L is the height of one of the triangles serving as a side of the pyramid. *L* is the distance you would climb if you climbed up the middle of one of the sides of the pyramid.

In the ***aL* Triangle** graph, you can see the height, apothem, and slant height, *L*, form a right triangle. This can be seen in the 3D graph as well. Since you have the height and can find the apothem, you can use the Pythagorean theorem to find the slant height, *L*.

24)Set the control values to *r* = 1, *h* = 1, *n* = 3.

25)Calculate the lateral surface area of the pyramid.

a) Check the **Perimeter** tab and note the value of *P*.

b) Check the **Apothem** tab and note the value of *a*.

c) Select the **Slant Height** tab and see how *a* and the Pythagorean theorem are used to find the slant height, *L*.

d) Make note of the slant height, *L*.

e) Calculate the lateral surface area = $\frac{1}{2}$*_____*_____ = _____

Total surface area is the lateral surface area **plus** the area of the base, or $S = \frac{1}{2}PL + B$.

26)Calculate the total surface area of the pyramid.

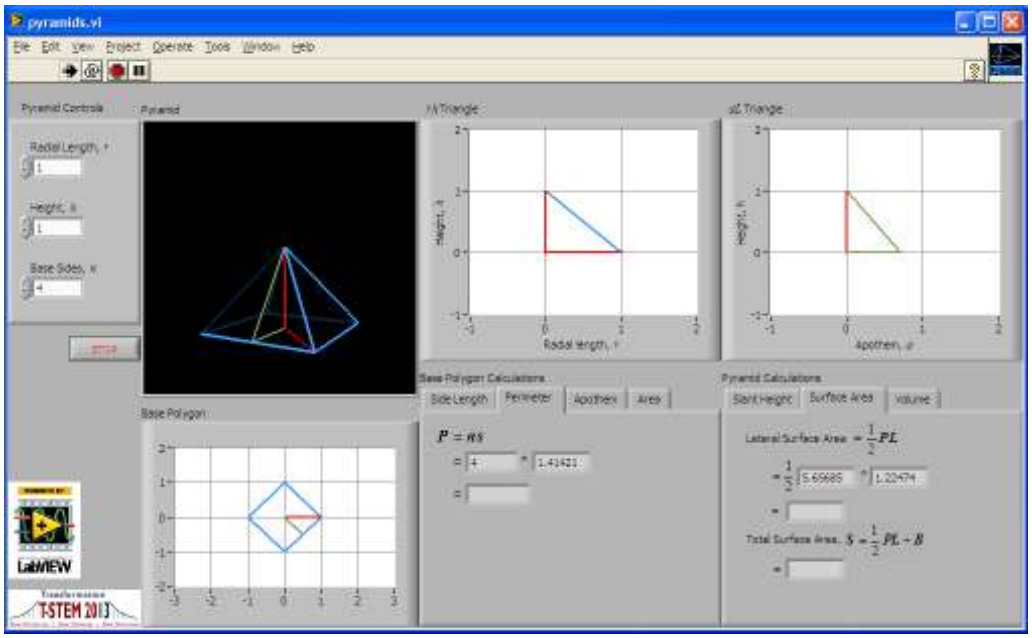
a) Check the **Area** tab and note the value of *B*.

b) Calculate the total surface area = $\frac{1}{2}$*_____*_____ + _____ = _____

27)Check your answers on the **Surface Area** tab.
28)Set the control values to *r* = 1.1547, *h* = 1, *n* = 3.
29)Calculate the total surface area of the pyramid.

a) Use VI to find *P*.
b) Use VI to find *L*.
c) Find lateral surface area = $\frac{1}{2}$*_____*_____ = _____
d) Use VI to find *B*.
e) Find *S* = $\frac{1}{2}$*_____*_____ + _____ = _____
f) Check your answers with the VI.

30)Set the control values to *r* = 1, *h* = 1, *n* = 4.



31)Calculate the total surface area of the pyramid.

a) Use VI to find *P*.
b) Use VI to find *L*.
c) Find lateral surface area = $\frac{1}{2}$*_____*_____ = _____
d) Use VI to find *B*.
e) Find *S* = $\frac{1}{2}$*_____*_____ + _____ = _____
f) Check your answers with the VI.

32)The perimeter of the base of a pyramid is 4 ft and the slant height is 3 ft.

a) Find the lateral surface area.
b) If the area of the base is 1 ft^2, what is the total surface area?

33) The length of one side of the triangular base of a pyramid is 1.5 cm and the slant height is 10 cm.
34) Find the lateral surface area.
35) If the area of the base is 5.433 cm^2, what is the total surface area?
36) What does the surface area tell you about a pyramid?
37) Stop the VI. You are done.