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T-STEM 2013
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CircleSphere Activity

Overview
Circles and spheres are all around us. Cups, bowls, moons, planets, orbits of satellites and the Doppler effect all exhibit circular or spherical characteristics. Circumference, area, surface area and volume are measurements related to these shapes giving us important information about the objects we make and use.

In this activity, students will learn how to find the diameter, circumference and area of a circle. They will learn how to find the surface area and volume of a sphere. They will develop an understanding of the derivation of some of the formulae used to calculate these values.

The CircleSphere virtual instrument, VI, will be used in this activity. The VI performs calculations to find our desired values as well as displaying images of a circle and a sphere for the given radius.

Objectives
Students will be able to:

- Calculate the diameter, circumference, and area of a circle with given radius
- Calculate the surface area and volume of a sphere with given radius

Standards (TEKS)
Geometry 2AB, 3B, 4, 5AB, 6AC, 7A, 8ADE, 9C, 10A

Activity
Circles

1) According to its definition, what is a **circle**?
2) Define the **radius** of a circle.

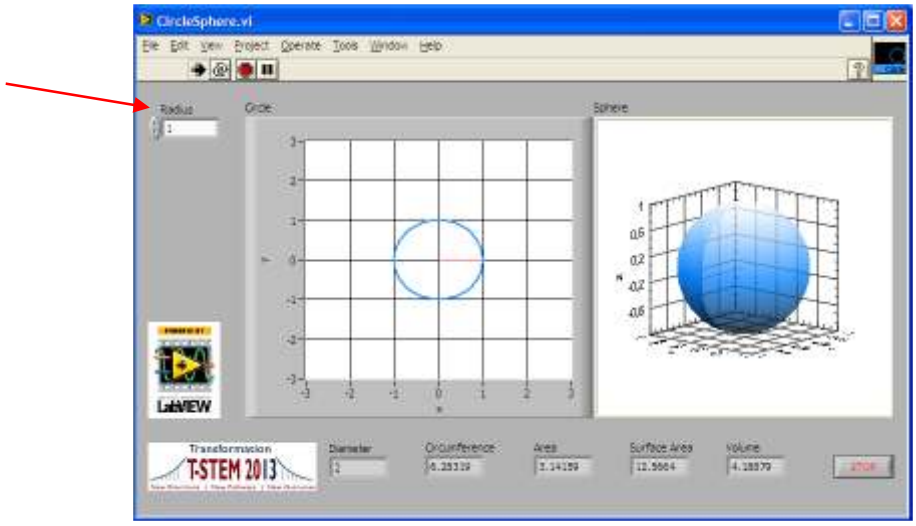
As in the first two questions, this activity will guide you to find and come up with definitions and formulas related to circles and spheres. If you get really stuck, there is a list of formulas on the back page. Don't peek yet. You may figure things out faster than you think.

Fill in the table by following the instructions in the questions below. Each row of the table corresponds to a circle with the given radius.

Radius, *r*	Diameter, *d*	Circumference, *C*	*C/d*	Area, *A*
1				
2				
3				

10				
100				

3) Open and run the CircleSphere virtual instrument, VI.
4) What is the **diameter** of a circle? How is related to the radius?
5) For each of the radii in the table, fill in the corresponding diameter. To do this with the VI,
    a) Use the **Radius** control to change the value of the radius.
    b) Notice how the graph of the circle changes as you change the radius.



The perimeter of, or distance around, a circle is called its **circumference**.

6) Use the VI to fill in the "Circumference" column of the table. Be sure to use the given radius values.
7) Do you see any pattern or rule for calculating the circumference? If so,
    a) Describe the rule or pattern.
    b) Use your rule to calculate the circumference of a circle whose radius is
        i. 5
        ii. 20
        iii. 11.5

If you haven't figured out a rule for circumference yet, don't worry. Next, you will take a close look at a famous number that will be of great help.

8) Find the column in the above table labeled "*C/d*"
    a) For each row, divide the circumference, *C*, of the circle by its diameter, *d*.
    b) Put your answers in the column labeled "*C/d*"
    c) What do you notice about your answers?

The answer you got each time you divided the circumference by the diameter is π.

9) In your own words, what is π?
10) Now, do you have a rule or formula for the circumference of a circle?
    a) If so, what is it?
    b) If not, start with the formula $\pi = \frac{C}{d}$, and solve for *C*.
11) Since you know the diameter is two times the radius, write another formula for circumference. This time have the formula in terms of *r* instead of *d.* (Hint: Substitute using $d = 2r$.)
12) Using one of your new formulas and a calculator, check the work you did using the VI when you filled in the "Circumference" column?
    a) Are there any discrepancies?
    b) If so, can you explain the differences?
13) Use a couple of Internet sites to find the radius of the Earth in kilometers.

    Radius: _____ Source: _____

    Radius: _____ Source: _____

    a) Do they answers agree? If not, why might they be different?
    b) Using one of the answers you found for the radius of the Earth, calculate the Earth's diameter in kilometers.
    c) Next, calculate the length of the equator. What units should be used?
14) What is the definition for the **area** of a circle?
15) If a circle's radius is given in meters, what units should be used when talking about the circle's area?
16) As you did with circumference, use the VI to fill in the "Area, *A*" column of the table. Be sure to use the given radius values.
17) Do you see any pattern or rule for calculating the area?
    a) If not, pay special attention to the area for circles of radius 1 and 2.
    b) Describe a rule or formula you might use to calculate area.
    c) Use your formula to calculate the area of a circle whose radius is
        i. 5
        ii. 20
        iii. 11.5
    d) Check your answers by using the VI.
        i. If they don't match, adjust your formula and try again.
        ii. If they do match, good job!

It is not uncommon to be asked for circumference and area in terms of π. "π" is the symbol used to refer to the exact value of the ratio of any circle's circumference to its diameter. π has infinitely many digits. Whether you have been using 3.14, or even

3.14159, as a value for π, it is an approximation. The exact circumference of circle with a radius of 1 ft is 2π ft. 6.28 is an approximation of 2π. Similarly, 9π ft^2 is the exact answer for the area of a circle whose radius is 3 ft.

18) In the table above, what did you put as the area of circle with a radius of 3?
19) What would the answer be in terms of π?
20) Fill in the following table with your answers in terms of π.

Radius, *r*	Diameter, *d*	Circumference, *C*	Area, *A*
1			
2			
5			

In the next part of the activity, you will take a look at spheres.

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Spheres

1) What is a **sphere**?

Fill in the table by following the instructions in the questions below. Each row of the table corresponds to a sphere with the given radius.

Radius, *r*	Surface Area, *S* (in terms of π)	Surface Area, *S* (approximate)	Volume, *V* (in terms of π)	Volume, *V* (approximate)
1				
2				
3				
10				
100				

If you had a piece of cloth that covered a ball completely with no overlaps, the area of the cloth when laid flat would be the **surface area** of the ball. The surface area of a sphere is the area of the "outside wall" of the sphere.

You can calculate the surface area of a sphere by using $S = 4\pi r^2$. As with the area of a circle, since the formula involves squaring the radius, the units for surface area are squared. That is, if the radius is in km, the surface area would be in km^2.

2) In the above table, fill in the first "Surface Area" column with the exact surface area of the sphere with the given radius.
    a. That is, put your answer in terms of π.
    b. For example, a sphere with a radius of 5 will have a surface area of $4\pi(5^2) = 4\pi(25) = 4(25)\pi = 100\pi$
3) Next, use a calculator and an approximation for π, like 3.14, to calculate approximate values for the surface areas.
    a. For example, if you use 3.14 to approximate π, the sphere with radius of 5 and surface area of 100π has an approximate surface area of 314.
    b. Fill the in the second "Surface Area" column in the above table with your approximations.
    c. Use the VI to check your answers. If they differ a bit, keep in mind the VI uses a better approximation of π than just 3.14.

In one of the questions in the *Circles* portion of this activity, you found an approximation for the radius of the Earth. You are going to use that approximation in a couple of the following questions. If you didn't do that part of the *Circles* portion, take a moment on the Internet to find two approximations for the radius of the Earth in kilometers. Pick the answer that comes from the more reliable source.

4) What is the surface area of the Earth? Be sure to label your answer with the correct units.
5) One the Internet, find an approximation for the radius of the moon.

   Radius: _____ Source: _____

6) What is the surface area of the moon? Check you units.
7) In your own words, what is the volume of a sphere?

You can calculate the surface area of a sphere by using $V = \frac{4}{3}\pi r^3$. Since the formula involves cubing the radius, the units for volume are cubed. That is, if the radius is in ft, the volume would be in ft^3.

8) In the above table, fill in the first "Volume" column with the exact volume of the sphere with the given radius.
   a. That is, put your answer in terms of π.
   b. For example, a sphere with a radius of 5 will have a volume,
   $$V = \frac{4}{3}\pi\left(5^3\right) = \frac{4}{3}\pi(125) = \frac{4}{3}(125)\pi = \frac{500}{3}\pi$$
9) Next, use a calculator and an approximation for π, like 3.14, to calculate approximate values for the surface areas.
   a. For example, if you use 3.14 to approximate π, the sphere with radius of 5 and volume of $\frac{500}{3}\pi$ has an approximate volume of 523.33.
   b. Fill the in the second "Volume" column in the above table with your approximations.
   c. Use the VI to check your answers. If they differ a bit, keep in mind the VI uses a better approximation of π than just 3.14.
10) What is the volume of the moon?
11) What is the volume of the Earth?
12) Compare the Earth's volume and surface area to the moon's.
   a. How many times larger is the volume of the Earth compared to the moon?
   b. How many times larger is the surface of the Earth compared to the moon?
   c. Are your answers what you expected?
      i. If so, explain what you expected.
      ii. If not, why not?
13) Stop the VI. You are done.

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Circle and Sphere Formulas

Diameter	$d = 2r$
Circumference	$C = 2\pi r$
Area	$A = \pi r^2$
Surface area	$S = 4\pi r^2$
Volume	$V = \frac{4}{3}\pi r^3$